

Mediterranean Youth Mathematical Championship (MYMC)

Rome, July 20, 2017

Late afternoon round – Single common problem

Evaluate the following sum:

$$\sum_{n=2}^{20} (4n^3 - 6n^2 + 4n) .$$

(The meaning is: the sum of the 19 numbers obtained from $(4n^3 - 6n^2 + 4n)$ by replacing n with 2, 3, 4, ..., 20.)

Solution

The answer is 160018.

To compute the sum, we can perform the following algebraic manipulations:

$$\begin{aligned} & \sum_{n=2}^{20} (4n^3 - 6n^2 + 4n) \\ &= \sum_{n=2}^{20} (n^4 - n^4 + 4n^3 - 6n^2 + 4n - 1 + 1) \\ &= \sum_{n=2}^{20} [n^4 - (n^4 - 4n^3 + 6n^2 - 4n + 1) + 1] \\ &= \sum_{n=2}^{20} [n^4 - (n-1)^4] + \sum_{n=2}^{20} 1 \\ &= 20^4 - 1^4 + 19 \\ &= 160018 . \end{aligned}$$