Mediterranean Youth Mathematical Championship (MYMC)
Trieste, July 8, 2015

Afternoon round – TIE-BREAKER

How many ways can we fill the cells of the following table with the colors blue, red, and green

11	12	13
21	22	23
31	32	33

such that no two adjacent cells contain the same color and it never happens that two numbers occurring in cells with the same color differ by 2 or by 20 (in absolute value)?

Solution

The answer is in *12* ways: *4* ways with the color B (blue) in the top left cell, *4* ways with the color R (red) in the top left cell, *4* ways with the color G (green) in the top left cell.
Let's see the case for color B, the other cases being analogous.
First step (four choices avoiding adjacent cells with same color):

B	R	
R		

B	R	
G		

B	G	
R		

B	G	
G		

Completion of the four tables forced by both rules:

B	R	G
R	G	B
G	B	R

B	R	G
G	B	R
R	G	B

B	G	R
R	B	G
G	R	B

B	G	R
G	R	B
R	B	G

For instance, in the first partial table the cell with 13 can only contain B or G (due to R to its left), but choosing B the first row would have two B cells numbered 11 and 13 (a violation of the second requirement), hence we are forced to put G there.