

# Numeri figurati ed equazioni diofantee

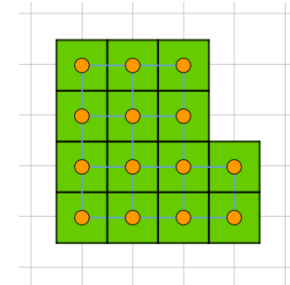
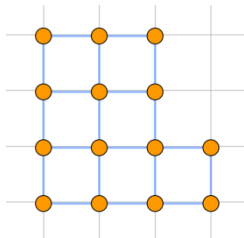
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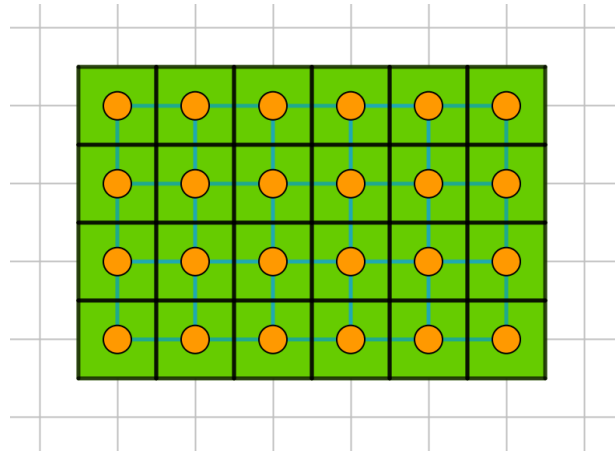
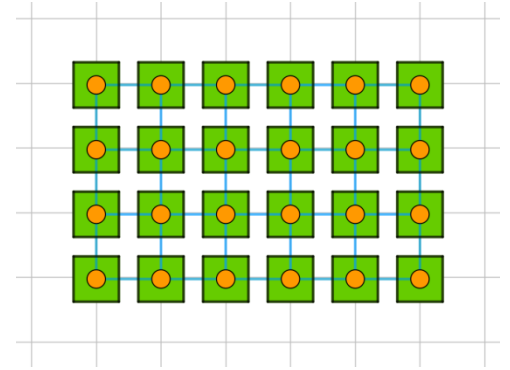
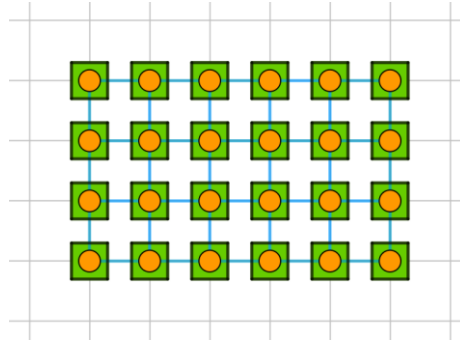
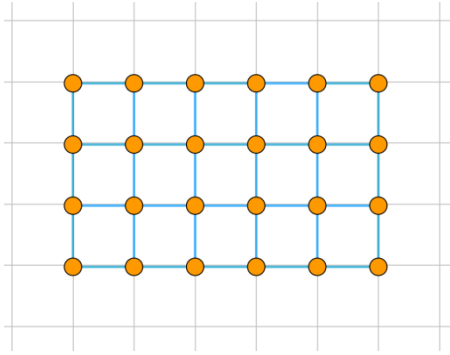
# Numeri figurati

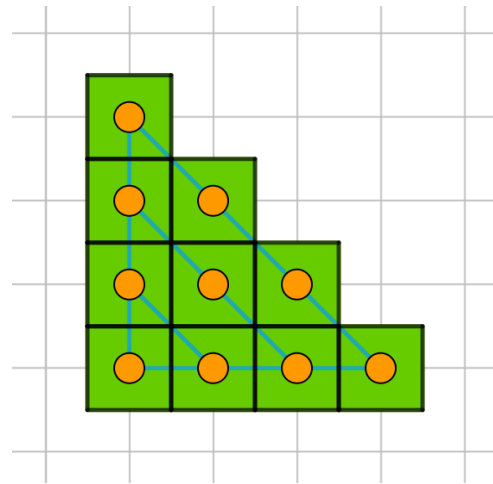
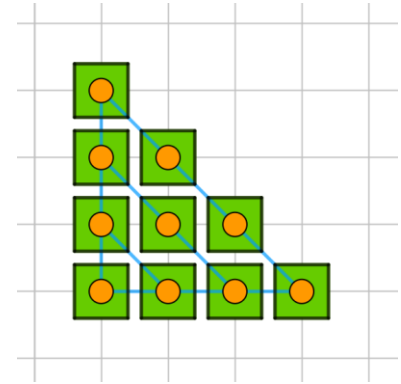
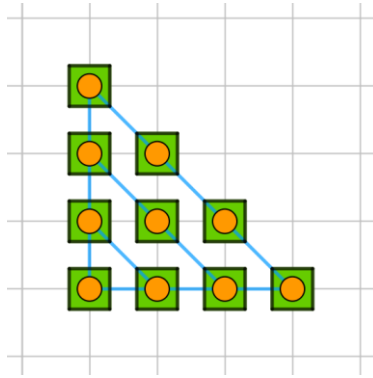
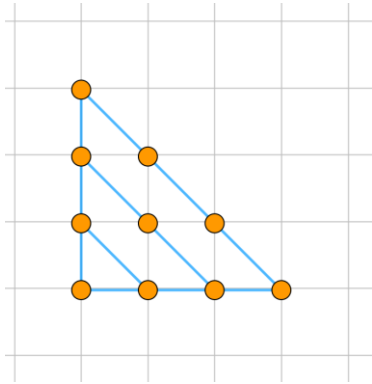
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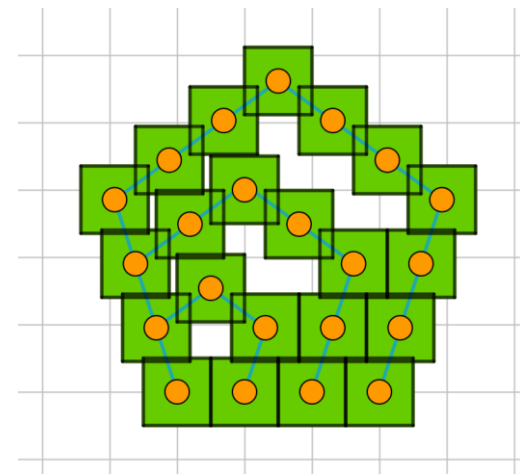
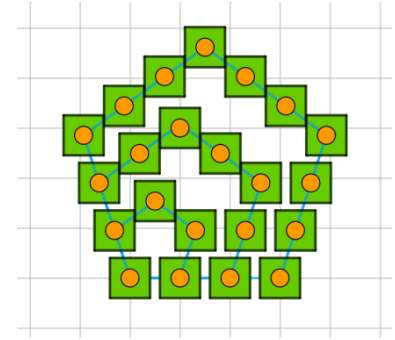
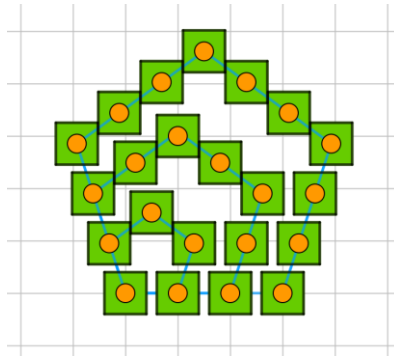
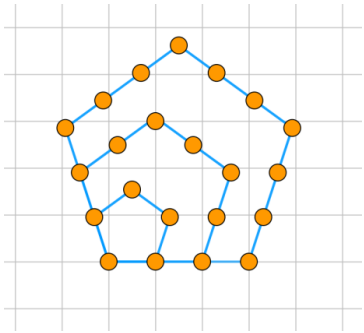
vs

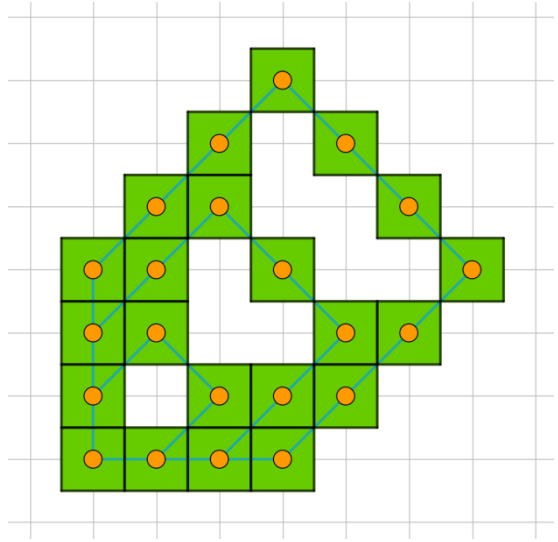
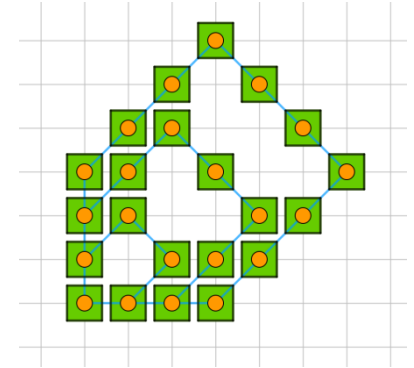
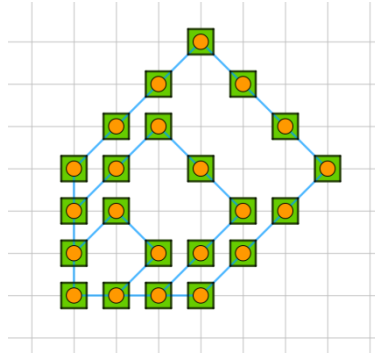
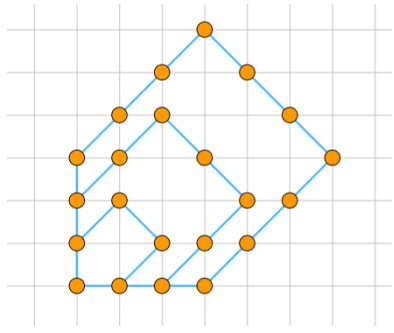
quadretti







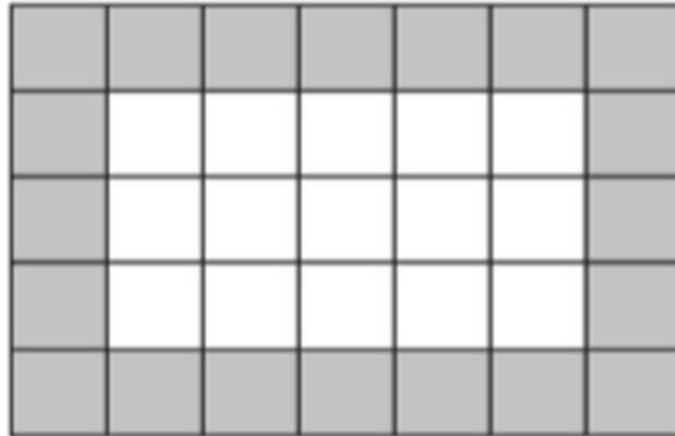




### **La terrazza**

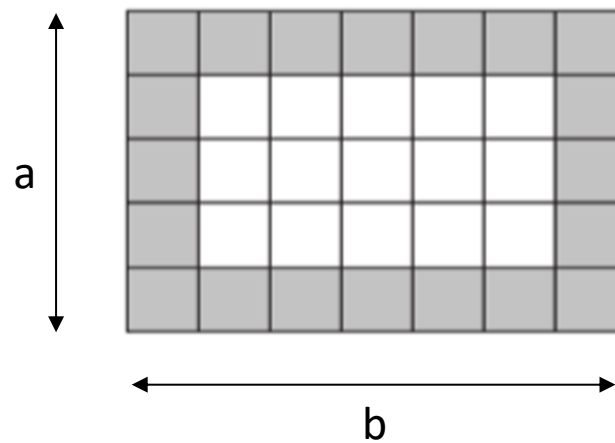
La terrazza di forma rettangolare che vedete in figura (le dimensioni non sono però indicative) è stata pavimentata con delle piastrelle grigie sui quattro lati del suo bordo. Le altre piastrelle, quelle interne, sono bianche.

**Quante sono al massimo le piastrelle grigie, sapendo che il loro numero è la metà di quello delle piastrelle bianche?**



Mattonelle bianche

Mattonelle grigie

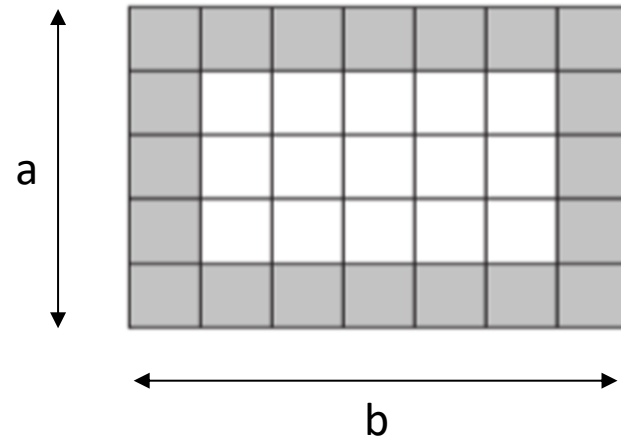






Mattonelle bianche  $(a - 2)(b - 2)$

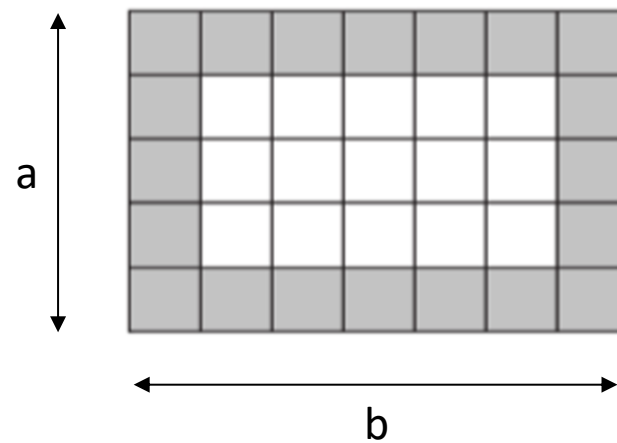
Mattonelle grigie  $2a + 2b - 4$



Mattonelle bianche  $(a - 2)(b - 2)$

Mattonelle grigie  $2a + 2b - 4$

$$(a - 2)(b - 2) = 2(2a + 2b - 4)$$

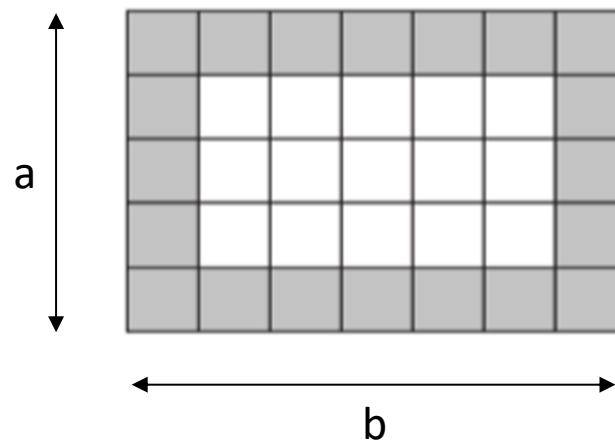


Mattonelle bianche  $(a - 2)(b - 2)$

Mattonelle grigie  $2a + 2b - 4$

$$(a - 2)(b - 2) = 2(2a + 2b - 4)$$

$$ab - 2a - 2b + 4 = 4a + 4b - 8$$



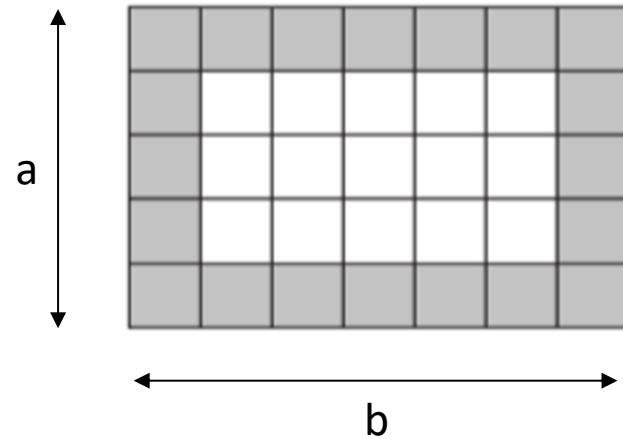
Mattonelle bianche  $(a - 2)(b - 2)$

Mattonelle grigie  $2a + 2b - 4$

$$(a - 2)(b - 2) = 2(2a + 2b - 4)$$

$$ab - 2a - 2b + 4 = 4a + 4b - 8$$

$$ab - 2a - 4a = 4b - 8 + 2b - 4$$



Mattonelle bianche  $(a - 2)(b - 2)$

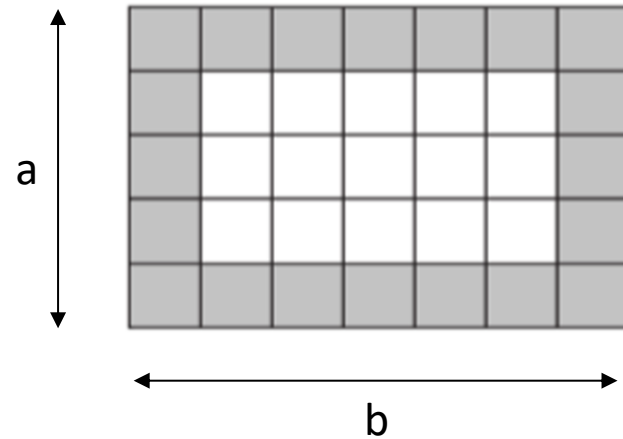
Mattonelle grigie  $2a + 2b - 4$

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$$ab - 2a - 2b + 4 = 4a + 4b - 8$$

$$ab - 2a - 4a = 4b - 8 + 2b - 4$$

$$ab - 6a = 6b - 12$$



Mattonelle bianche  $(a - 2)(b - 2)$

Mattonelle grigie  $2a + 2b - 4$

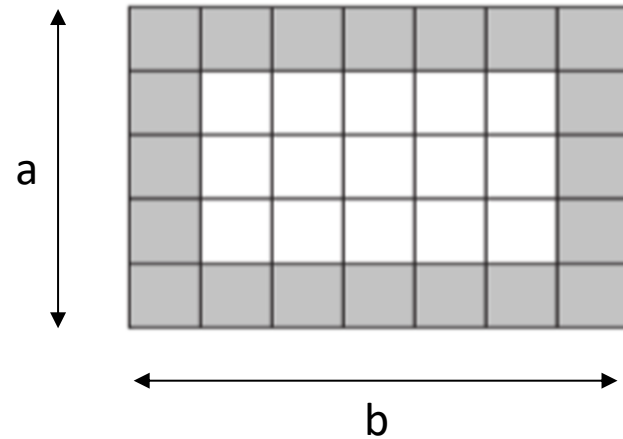
$$(a - 2)(b - 2) = 2(2a + 2b - 4)$$

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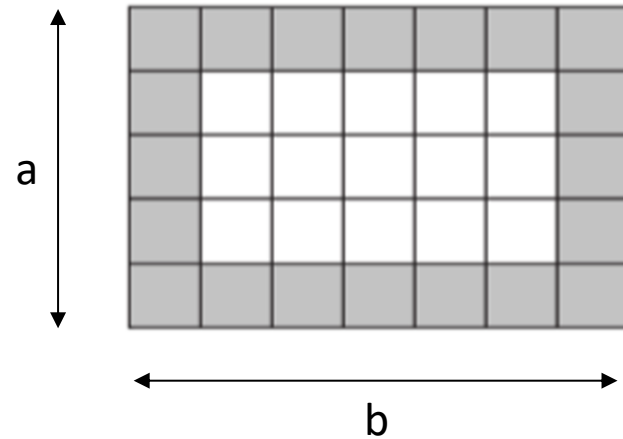
$$ab - 2a - 2b + 4 = 4a + 4b - 8$$

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$$ab - 6a = 6b - 12$$

$$a(b - 6) = 6b - 12$$

$$b \neq 6$$





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Mattonelle grigie  $2a + 2b - 4$

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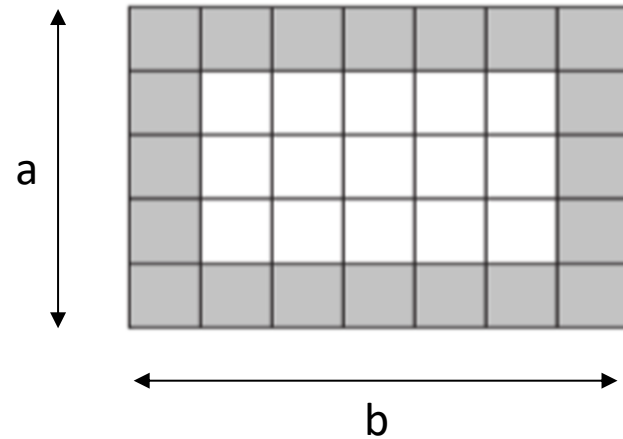
$$ab - 2a - 4a = 4b - 8 + 2b - 4$$

$$ab - 6a = 6b - 12$$

$$a(b - 6) = 6b - 12$$

$$b \neq 6$$

$$a = \frac{6b - 12}{b - 6}$$



Mattonelle bianche  $(a - 2)(b - 2)$

Mattonelle grigie  $2a + 2b - 4$

$$(a - 2)(b - 2) = 2(2a + 2b - 4)$$

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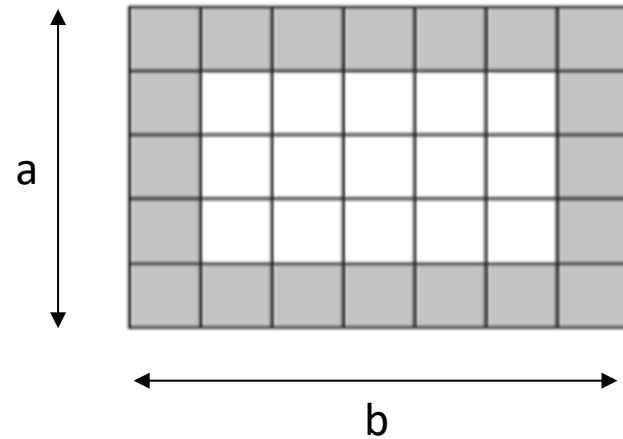
$$ab - 6a = 6b - 12$$

$$a(b - 6) = 6b - 12$$

$$b \neq 6$$

$$a = \frac{6b - 12}{b - 6}$$

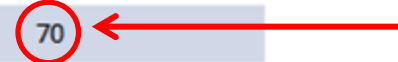
$$a = 6 + \frac{24}{b - 6}$$



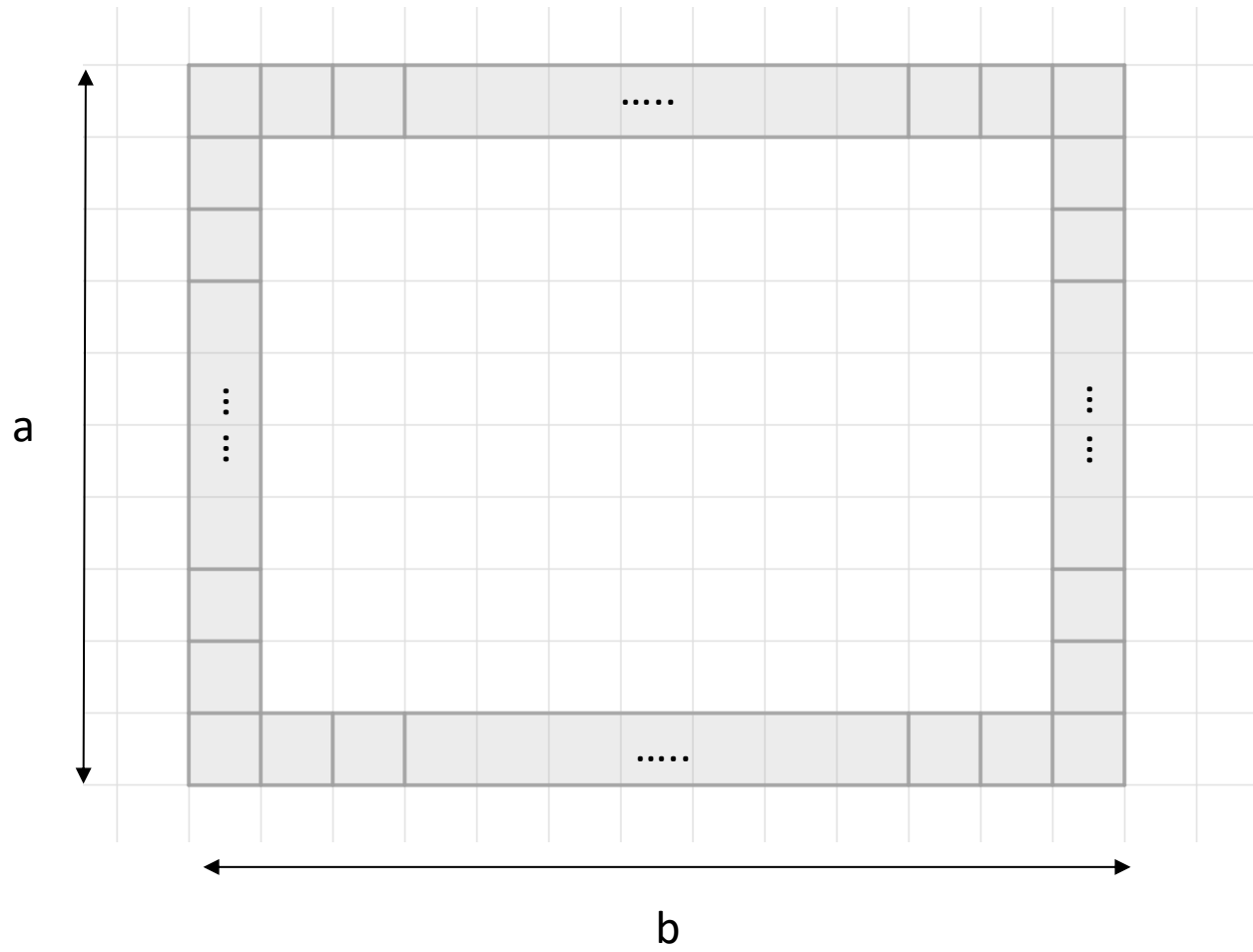
$b - 6$	$\frac{24}{b - 6}$	$a$	$b$	Mattonelle bianche $(a - 2)(b - 2)$	Mattonelle grigie $2a + 2b - 4$
1	24	30	7	140	70
2	12	18	8	96	48
3	8	14	9	84	42
4	6	12	10	80	40
6	4	10	12	80	40
8	3	9	14	84	42
12	2	8	18	96	48
24	1	7	30	140	70
-1	-24	-18	5	-60	-30
-2	-12	-6	4	-16	-8
-3	-8	-2	3	-4	-2
-4	-6	0	2	0	0
-6	-4	2	0	0	0
-8	-3	3	-2	-4	-2
-12	-2	4	-6	-16	-8
-24	-1	5	-18	-60	-30

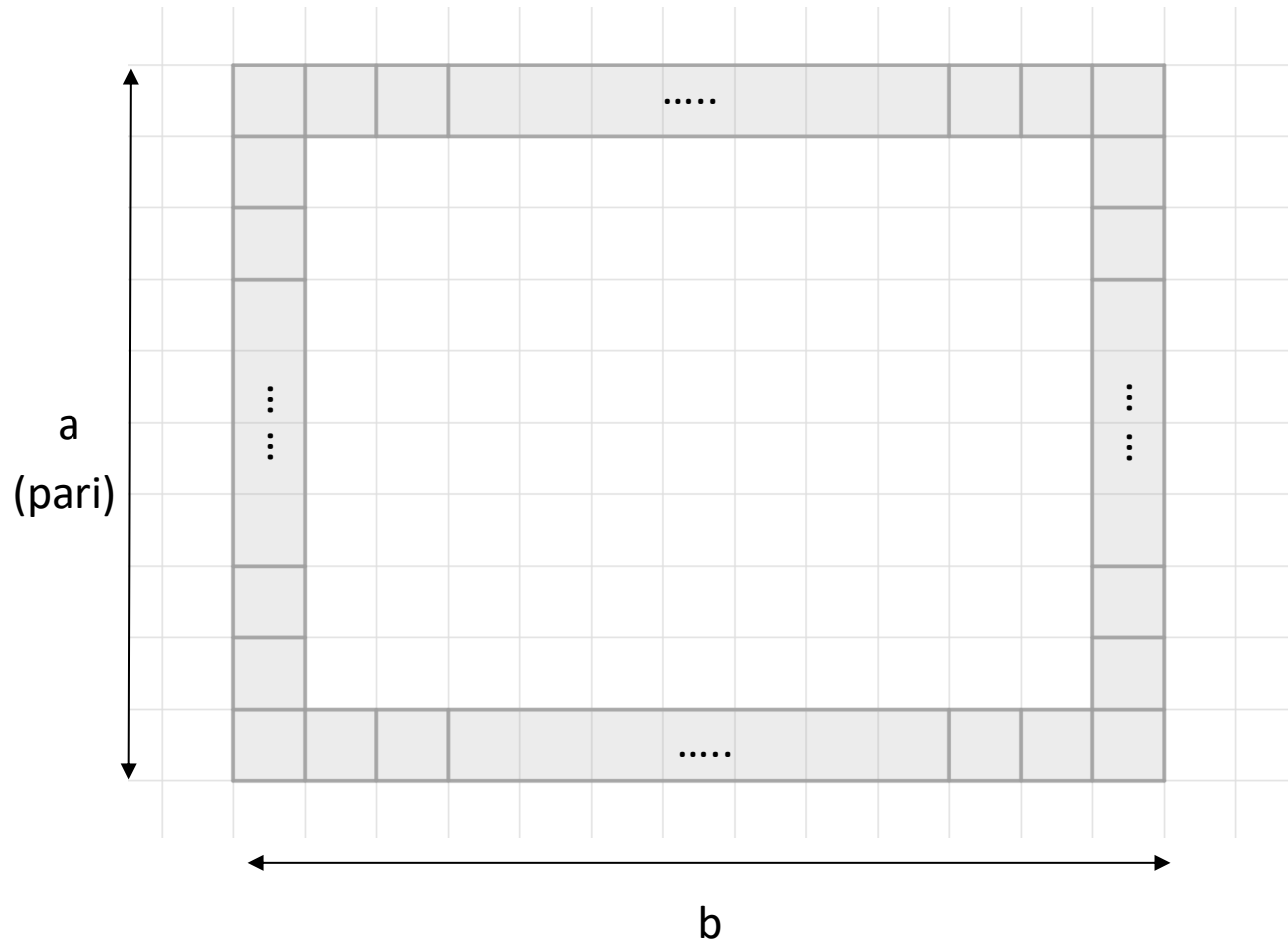
$$a = 6 + \frac{24}{b - 6}$$

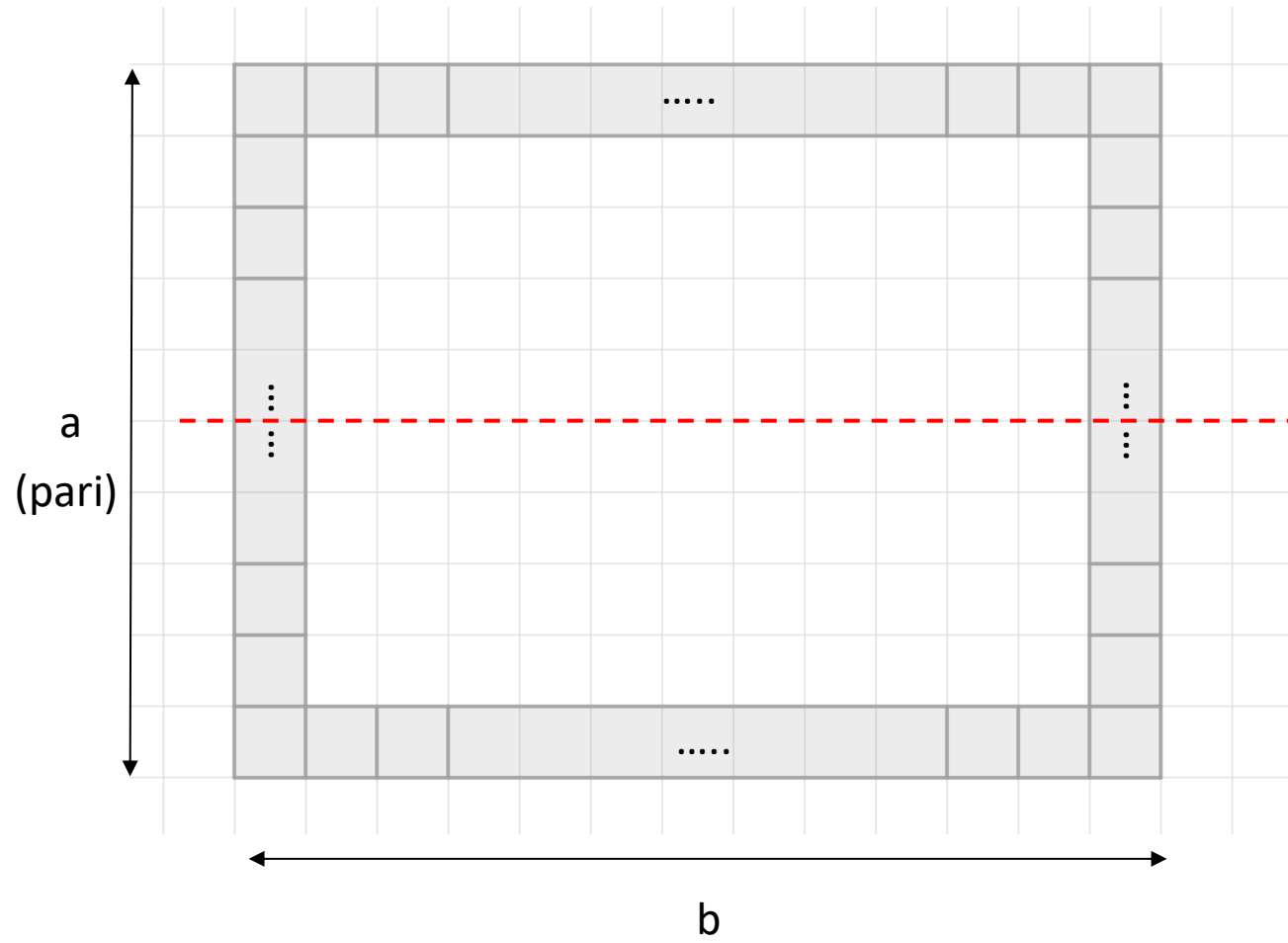
$b - 6$	$\frac{24}{b - 6}$	$a$	$b$	Mattonelle bianche $(a - 2)(b - 2)$	Mattonelle grigie $2a + 2b - 4$
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-6	-4	2	0	0	0
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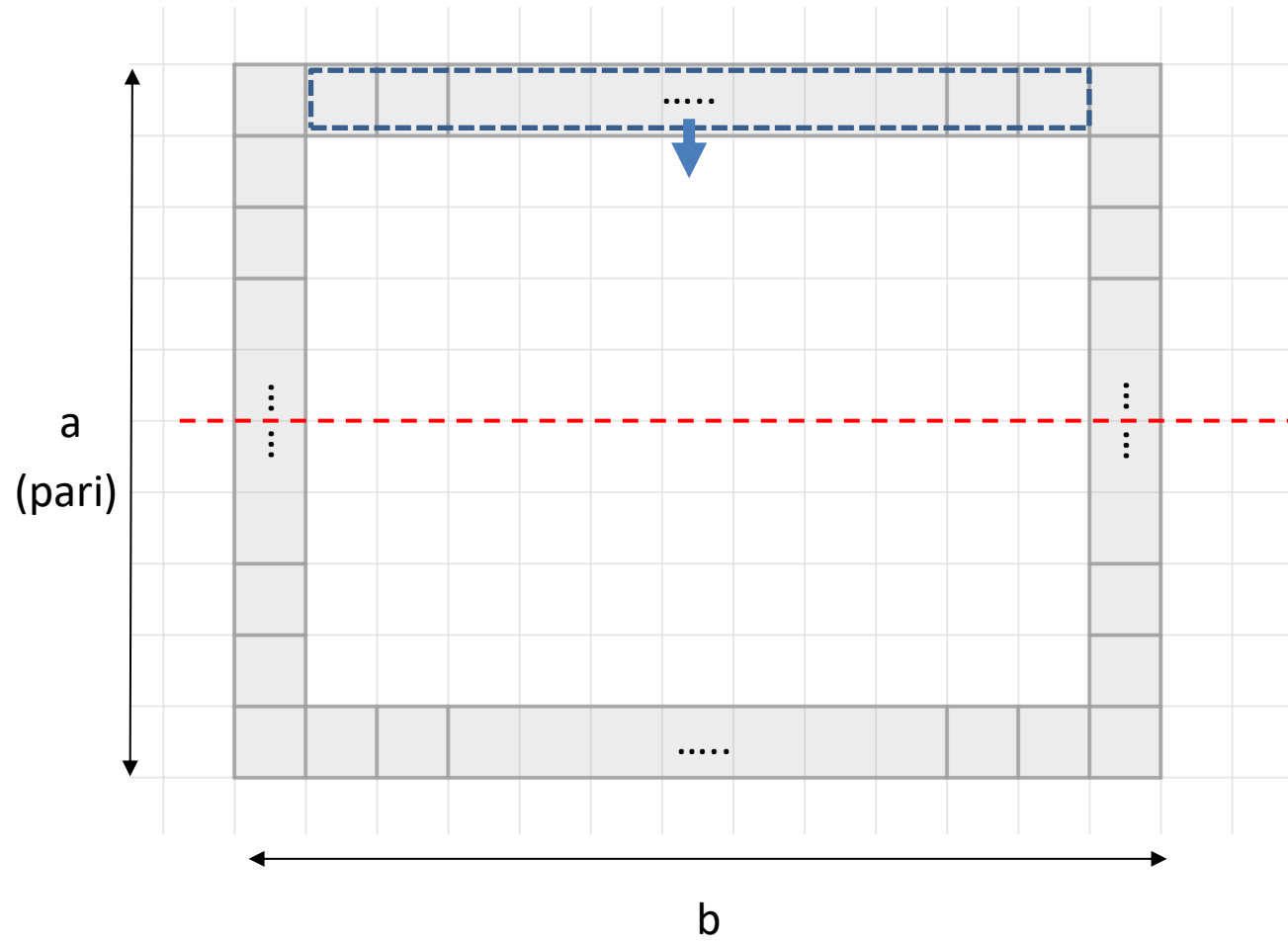


$$a = 6 + \frac{24}{b - 6}$$

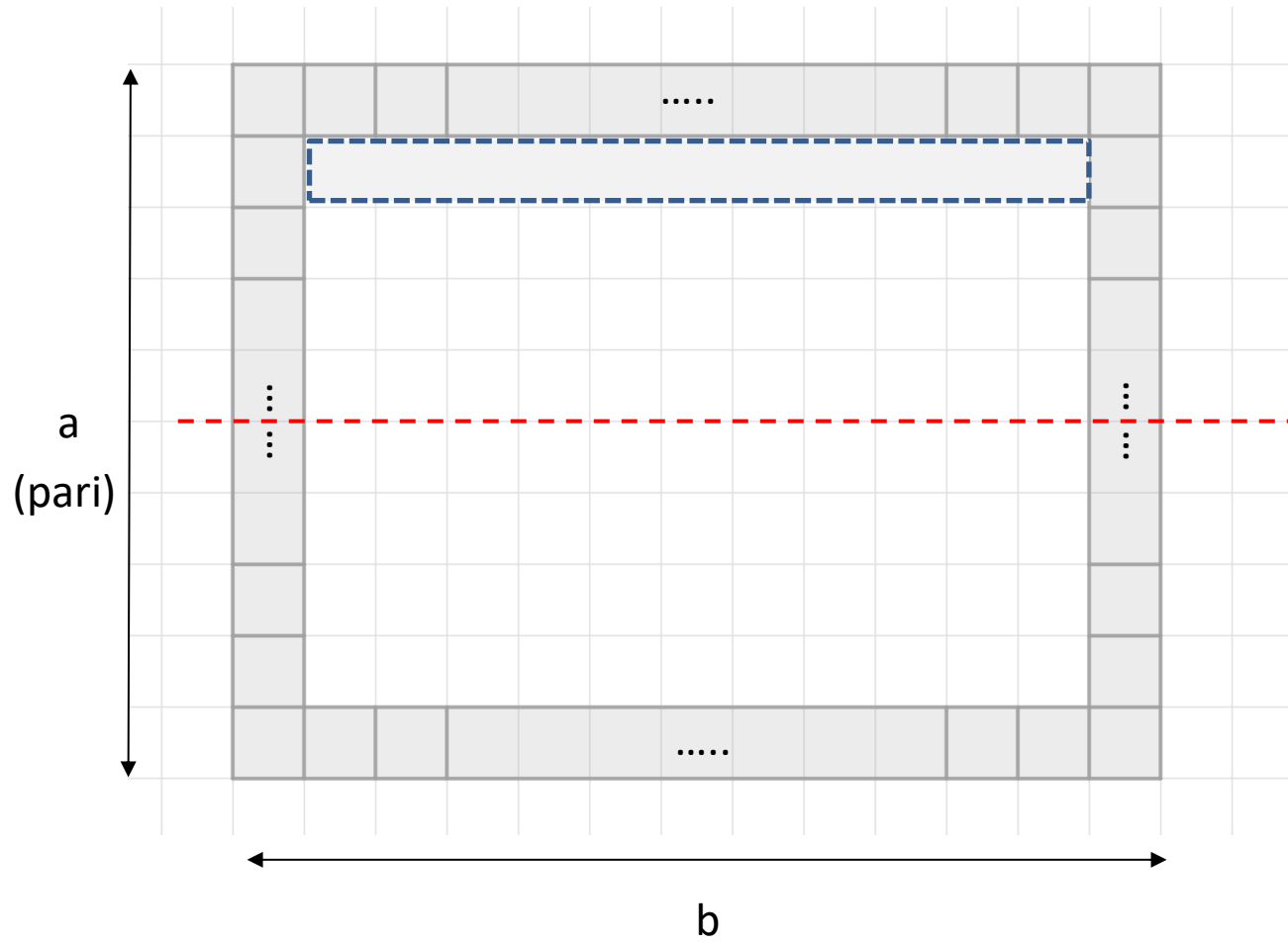


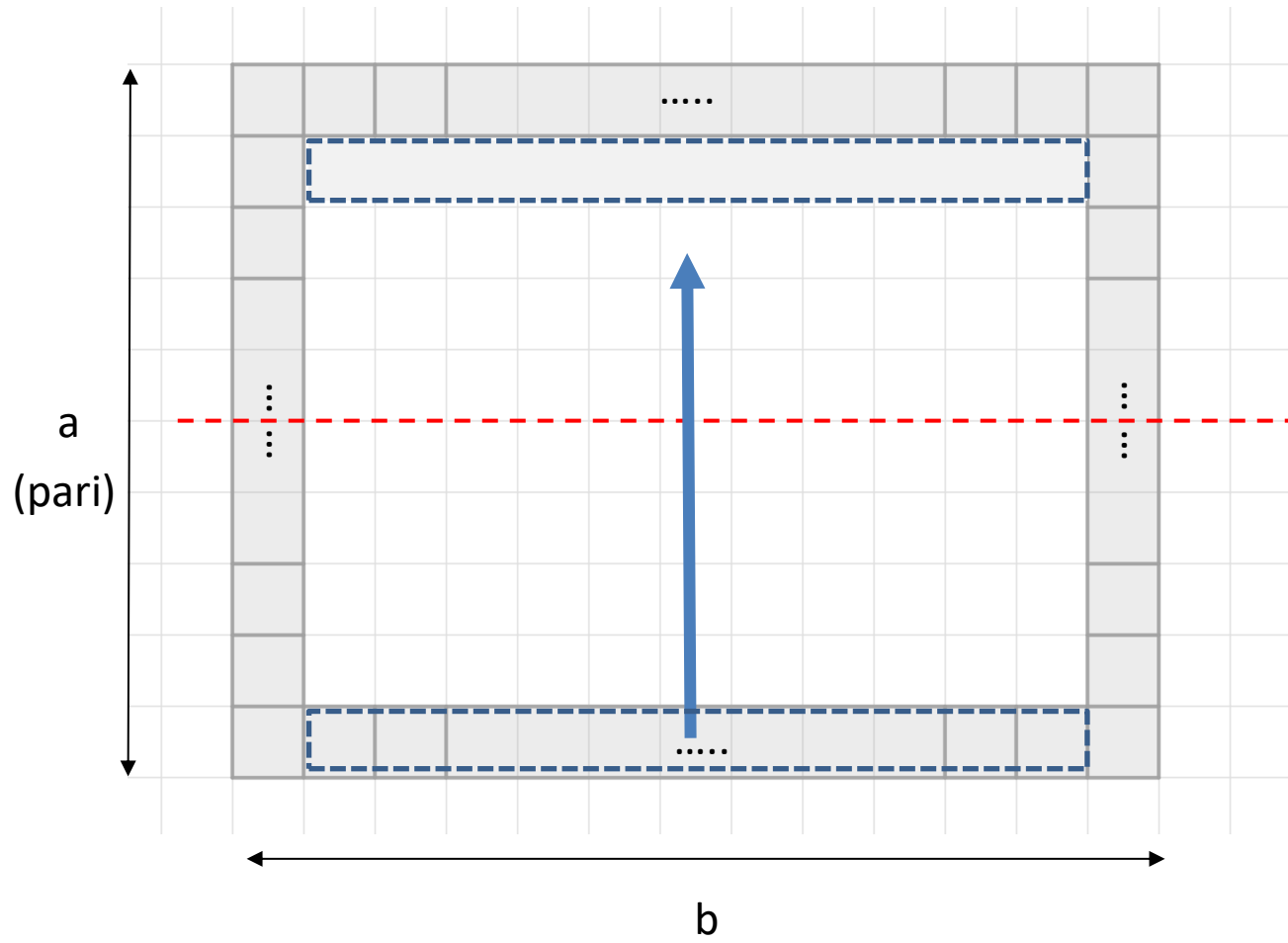


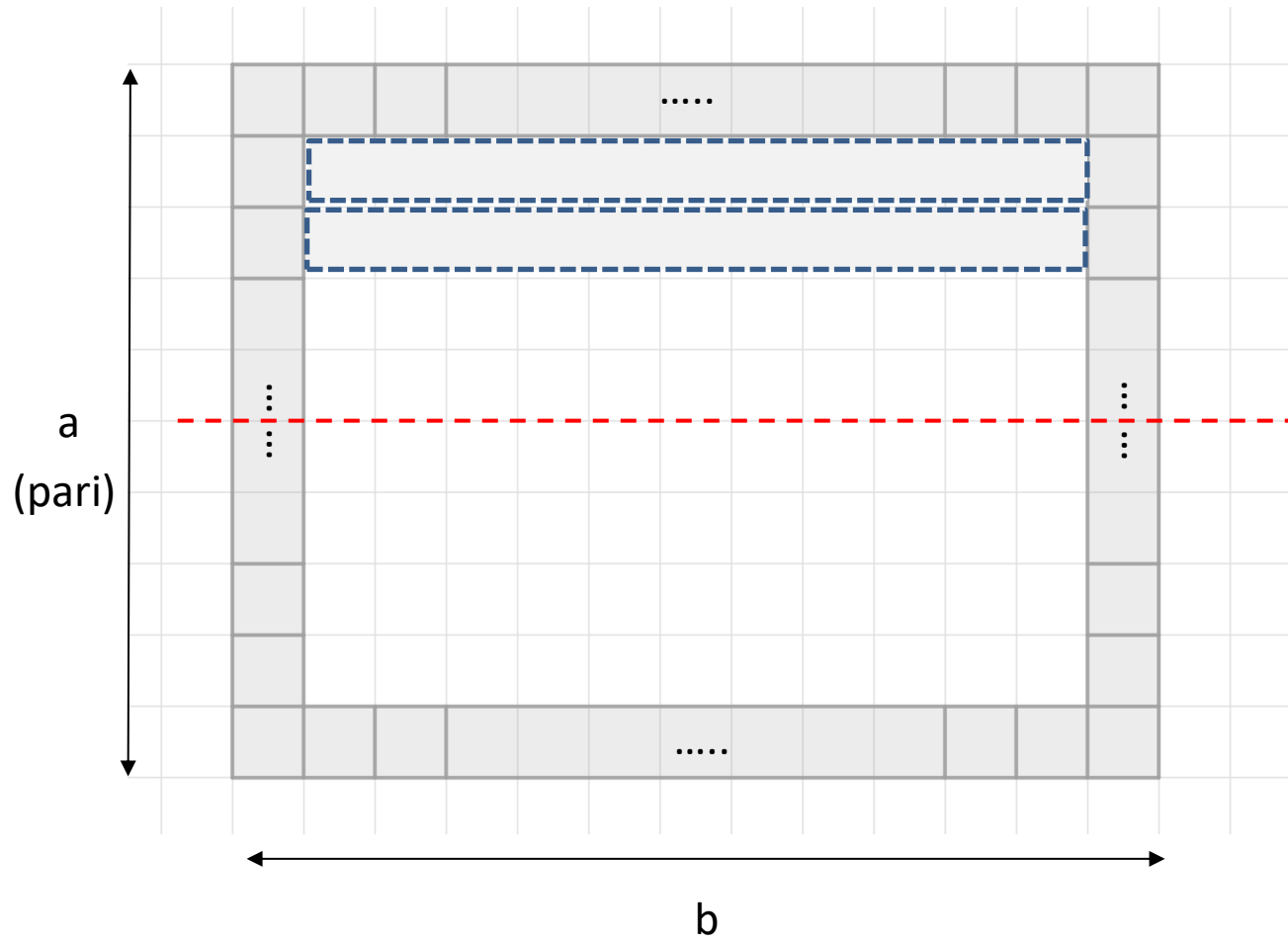




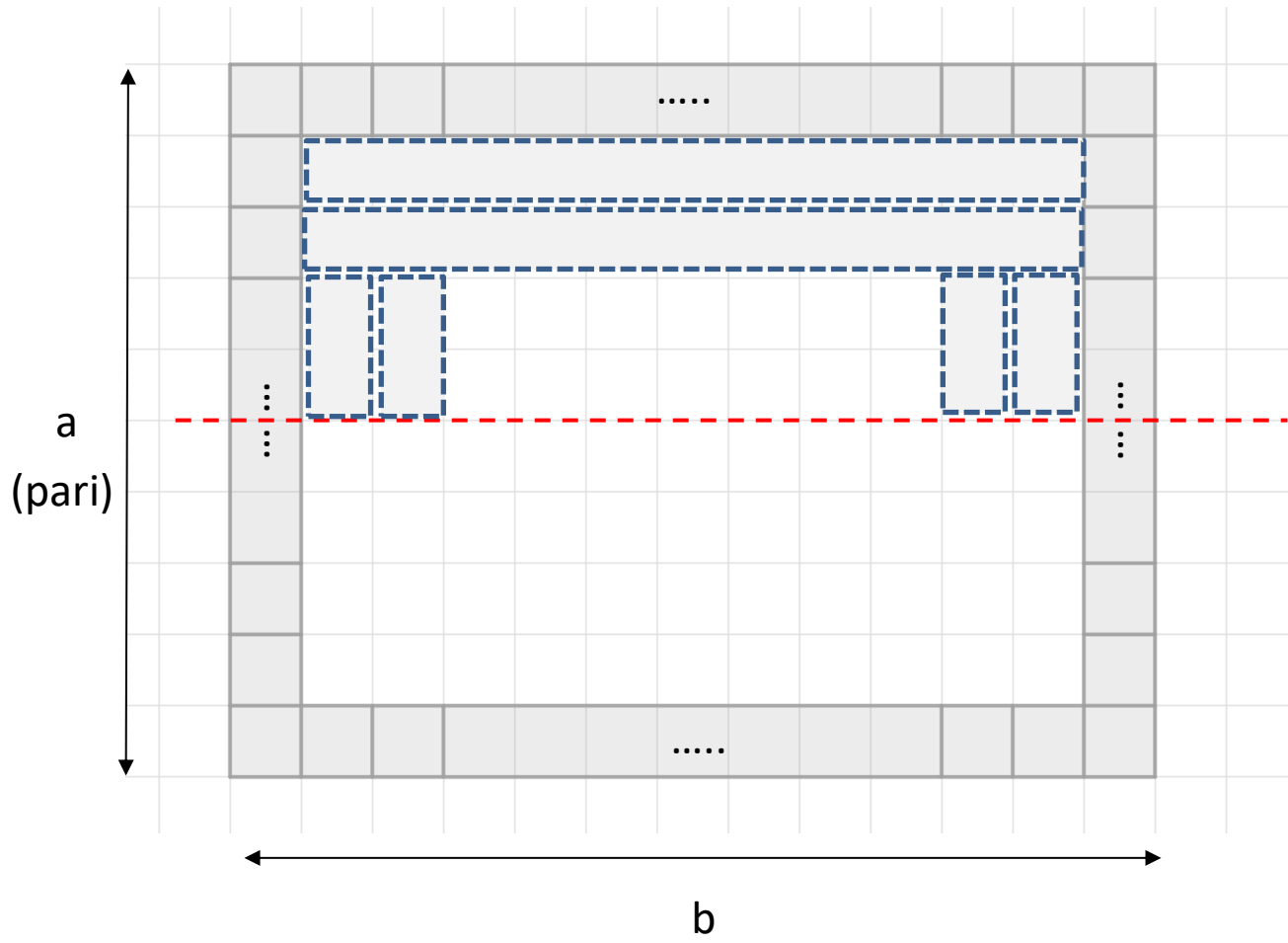




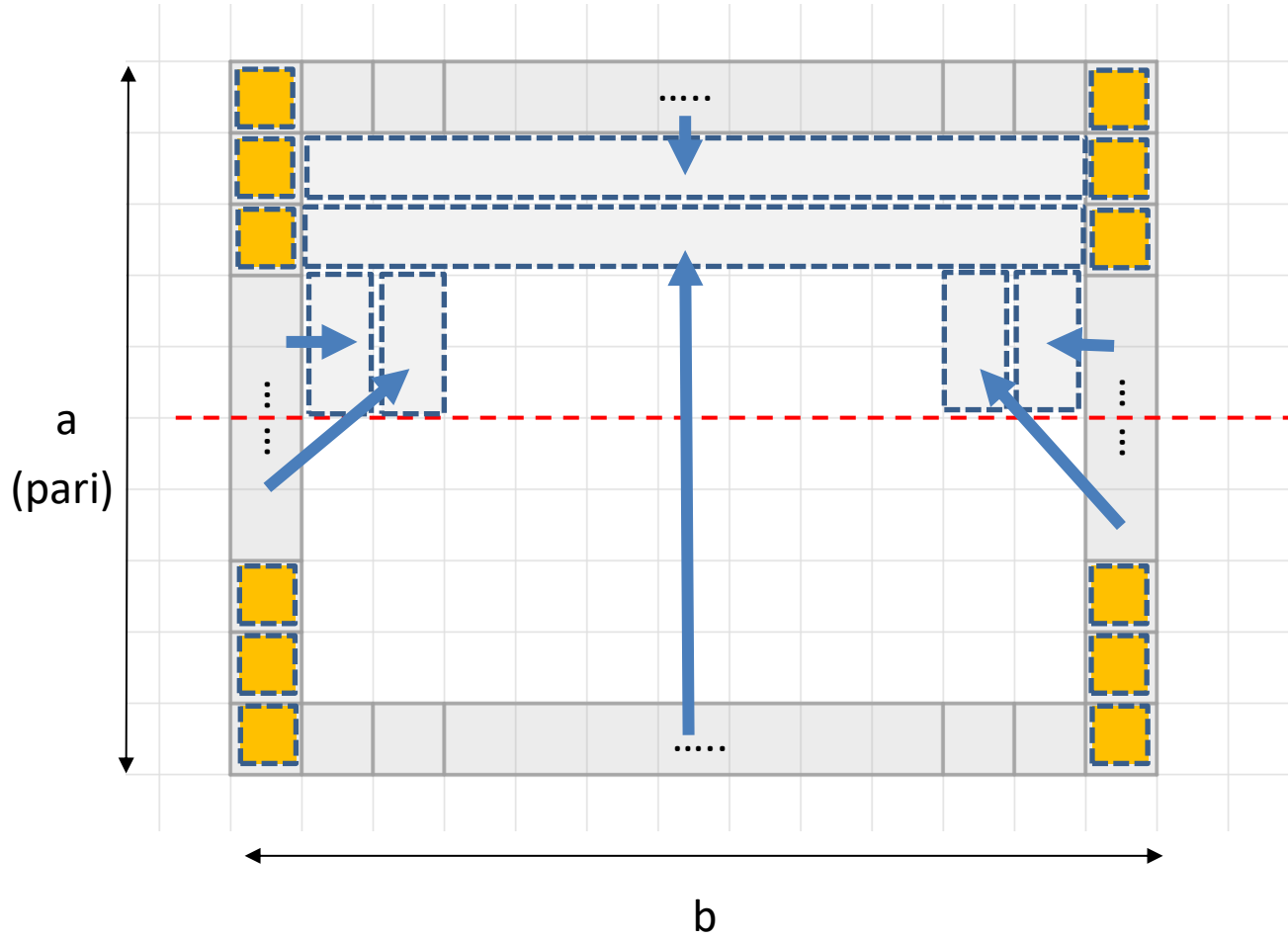


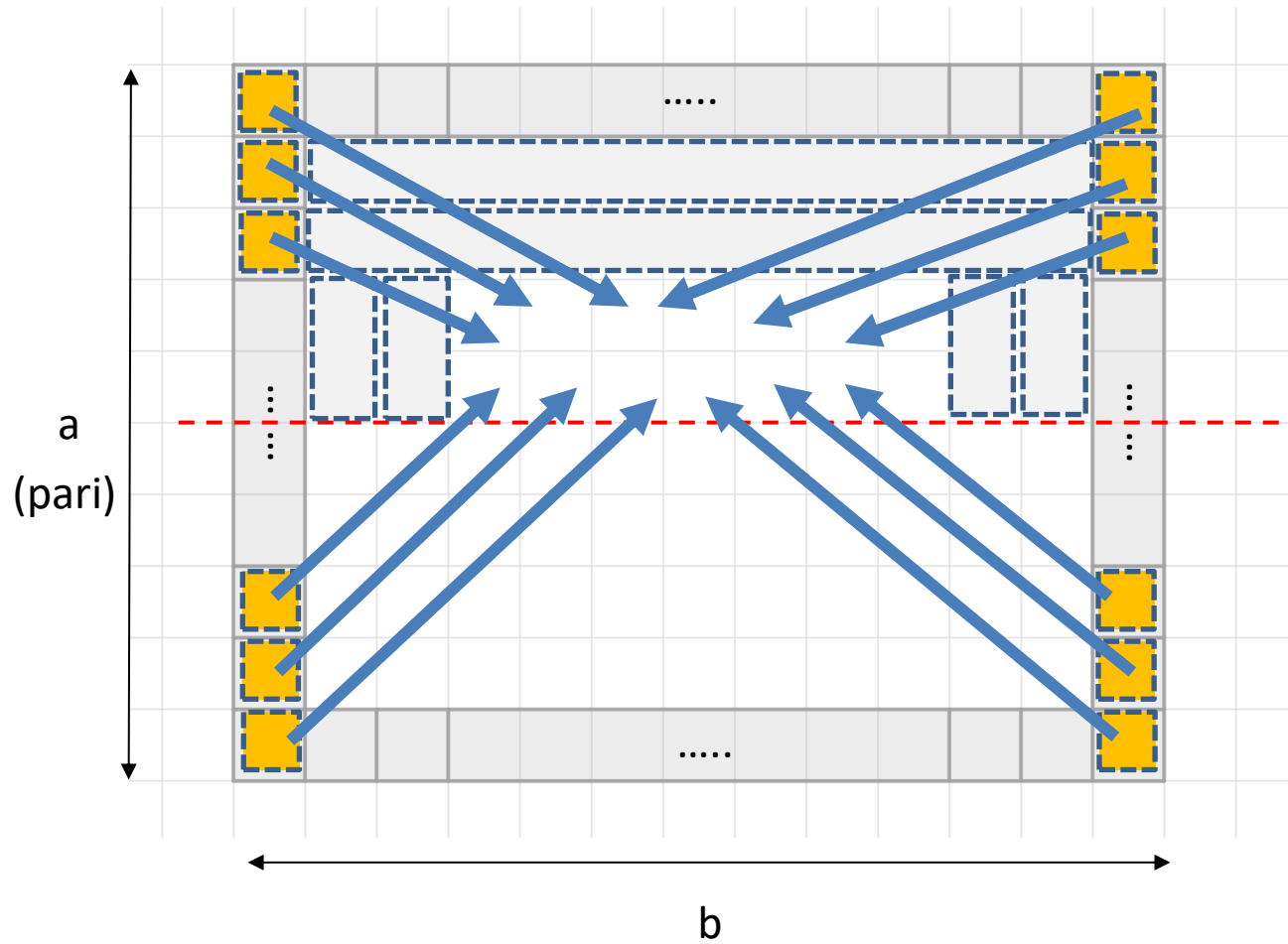




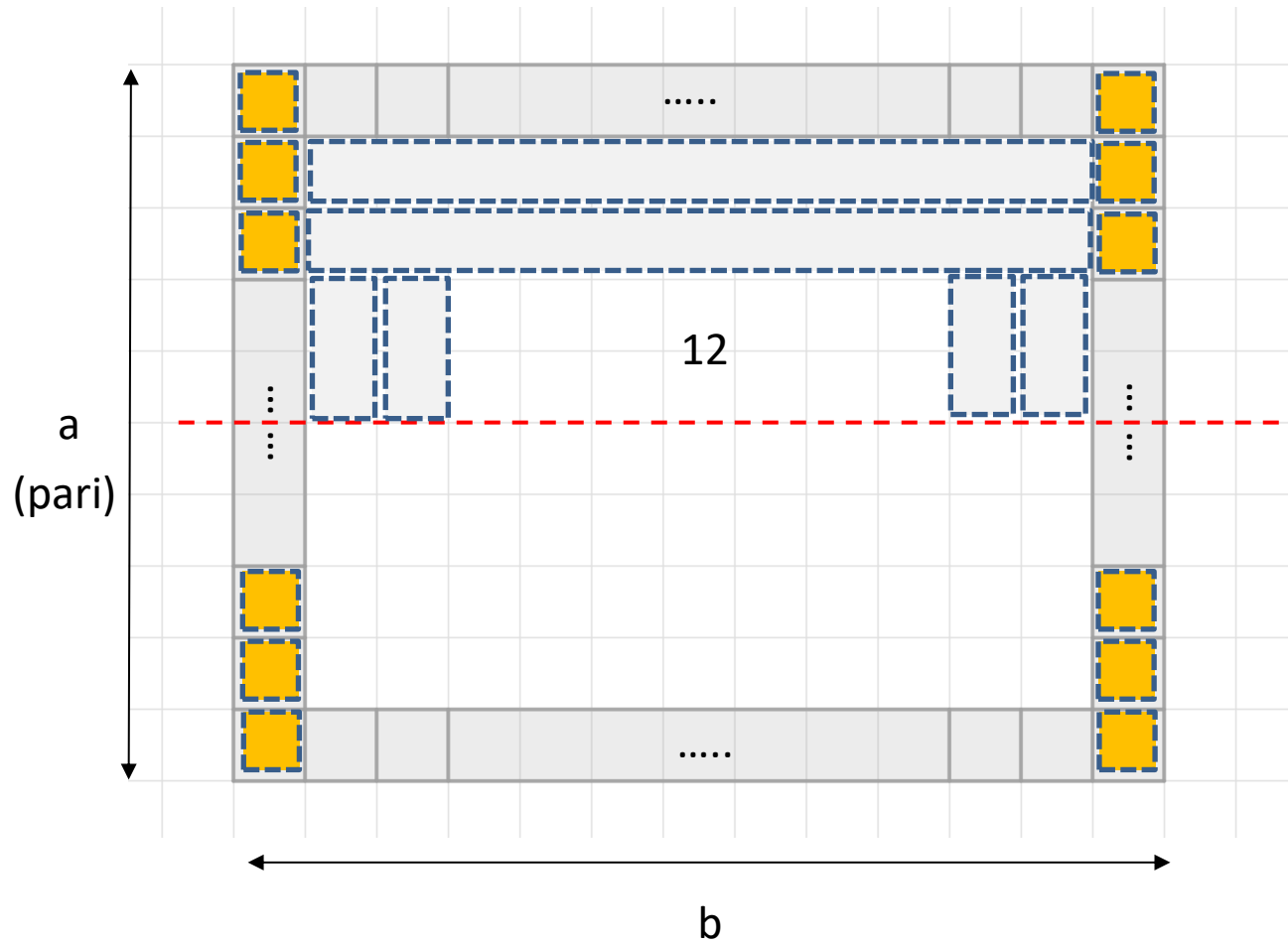


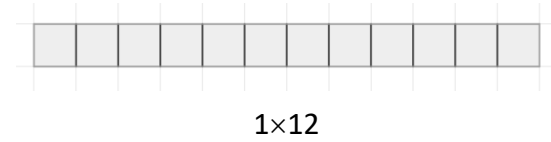
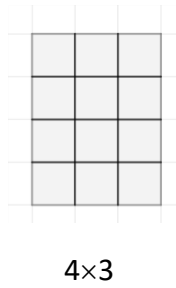
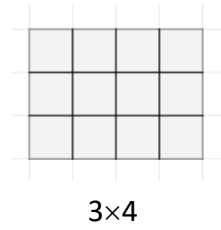
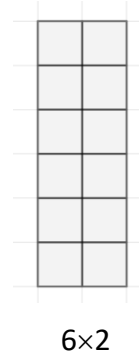
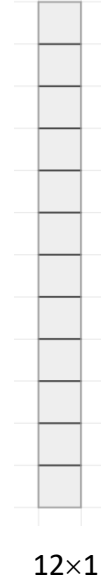
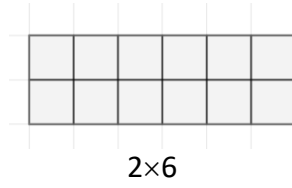
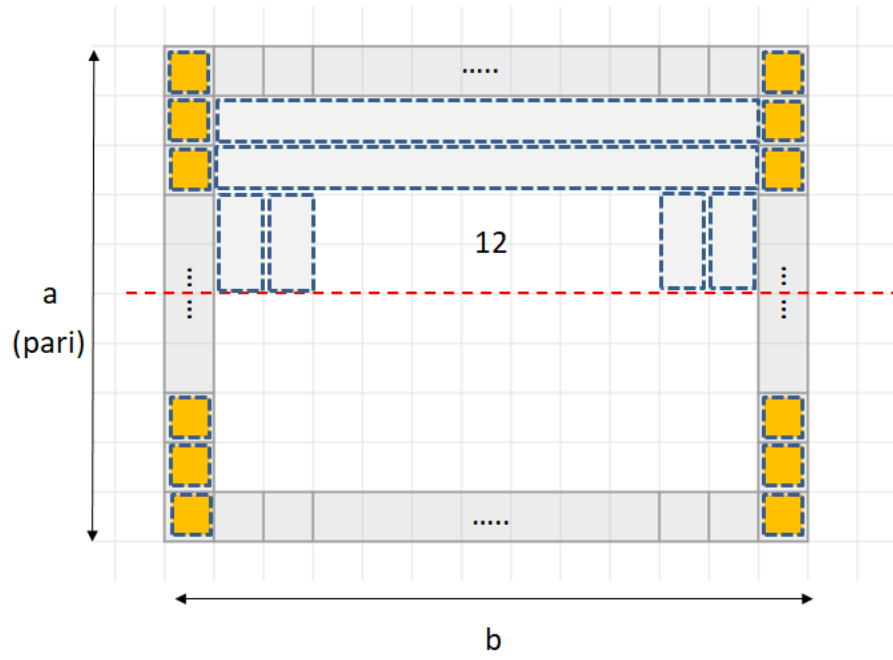


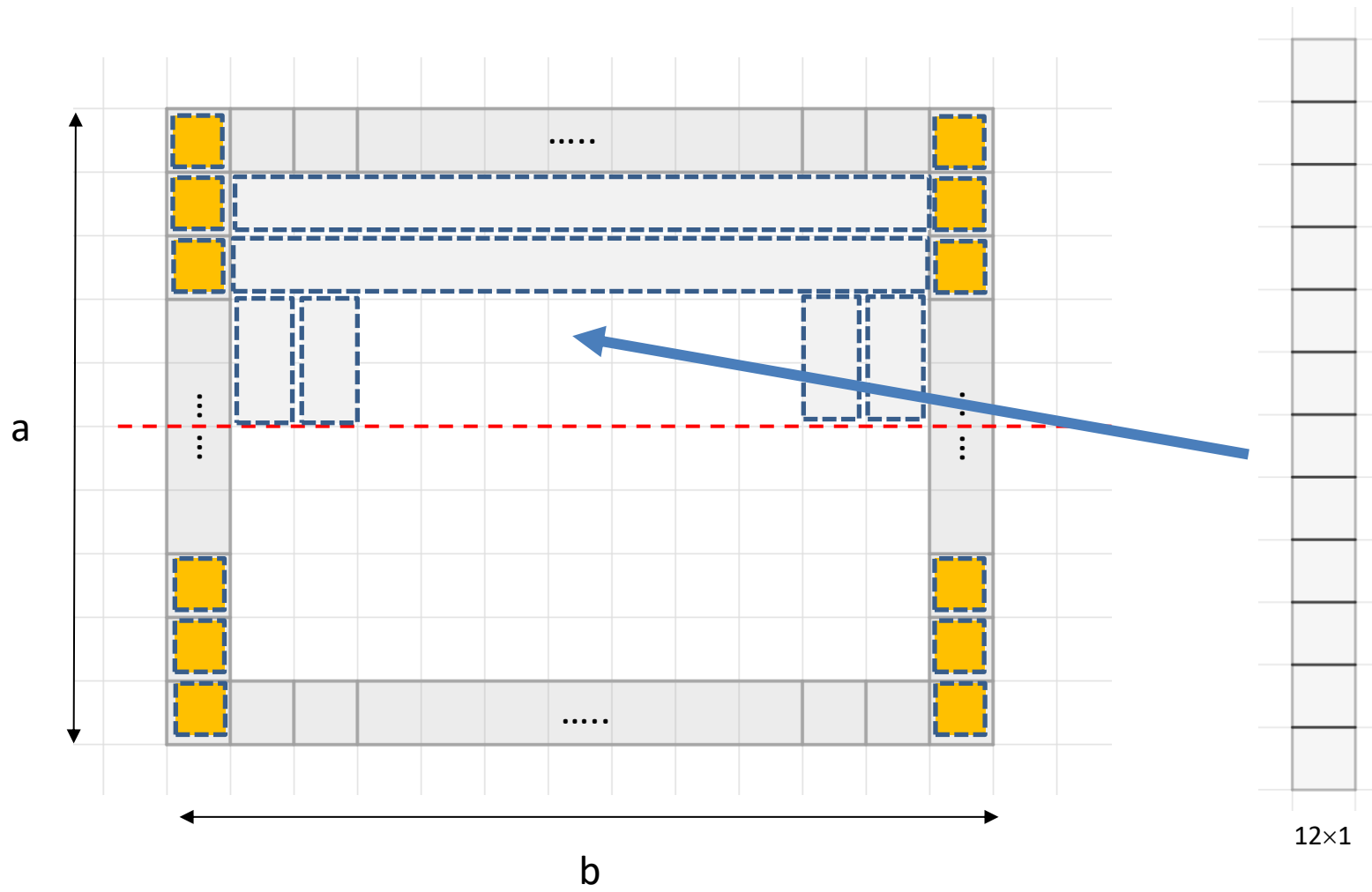




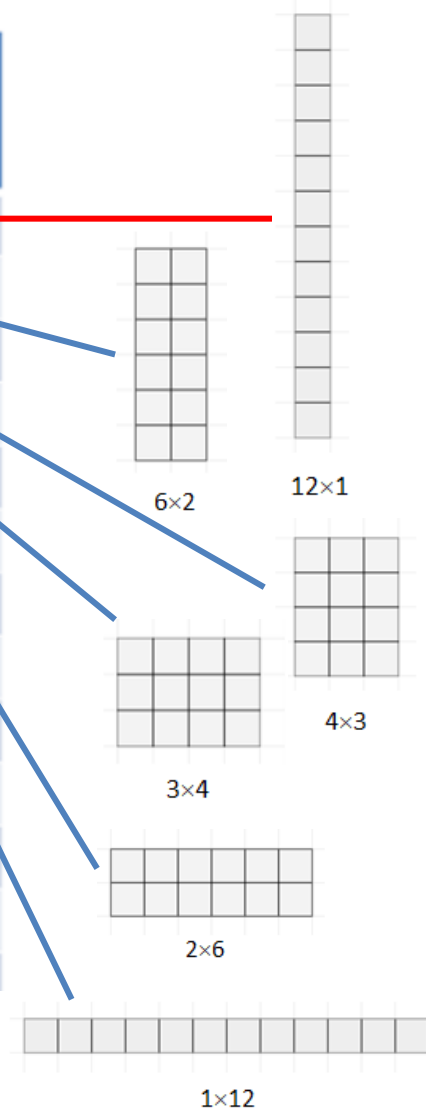








$b - 6$	$\frac{24}{b - 6}$	$a$	$b$	Mattonelle bianche $(a - 2)(b - 2)$	Mattonelle grigie $2a + 2b - 4$
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-6	-4	2	0	0	0
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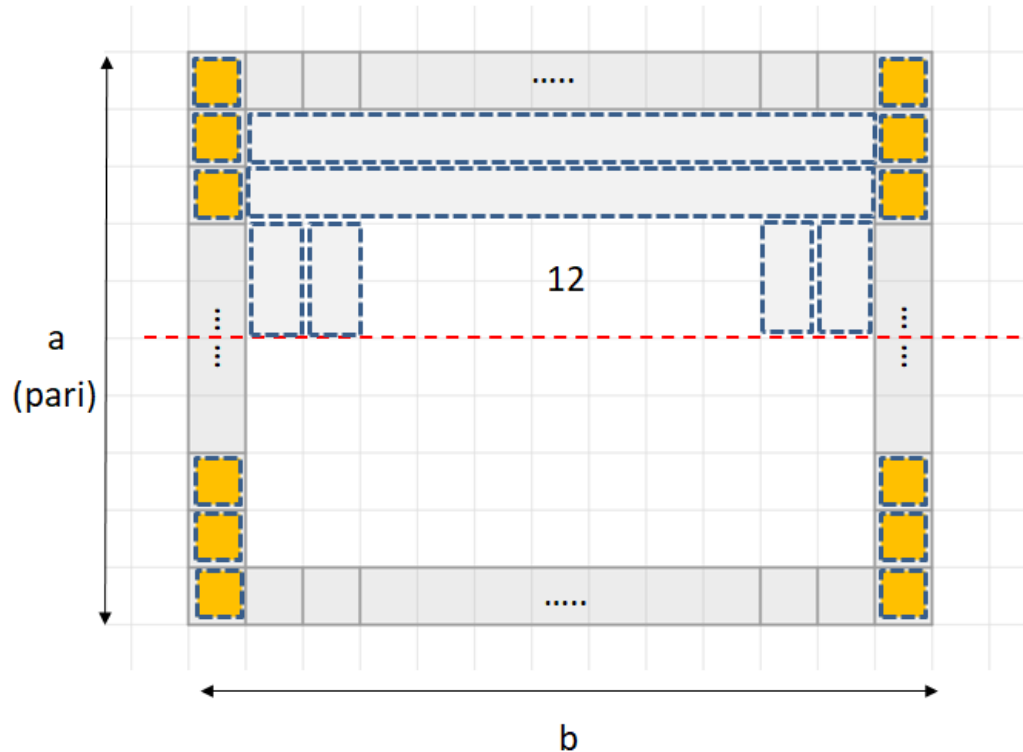
# Alcune osservazioni finali

- Le sei rappresentazioni rettangolari del 12 corrispondono alle righe con a pari

$b - 6$	$\frac{24}{b - 6}$	$a$	$b$	Mattonelle bianche $(a - 2)(b - 2)$	Mattonelle grigie $2a + 2b - 4$
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-24	-1	5	-18	-60	-30

## Alcune osservazioni finali

- Le sei rappresentazioni rettangolari del 12 corrispondono alle righe con  $a$  pari
- Non esistono soluzioni con  $b \leq 6$  perché altrimenti «mancherebbe» lo spazio per le dodici piastrelle arancio.



Grazie per l'attenzione!